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ENTIRE FUNCTIONS SHARING A LINEAR POLYNOMIAL WITH HIGHER ORDER DERIVATIVES OF LINEAR DIFFERENTIAL POLYNOMIAL

GOUTAM KUMAR GHOSH

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Abstract. In the paper we study the uniqueness of entire functions sharing a linear polynomial with higher order derivatives of linear differential polynomials generated by them. The results of the paper improve and generalize the corresponding results of Lahiri-Kaish (J. Math. Anal. Appl. 406(2013), 66–74).

2010 AMS Subject Classifications: 30D35

Key words and phases: Entire function, Linear Differential Polynomial, Uniqueness.

1. Introduction, Definitions and Results. Let f be a nonconstant meromorphic function in the open complex plane $\mathbb{C}$ and a be a polynomial. We denote $E(a;f)$ the set of a-points of f, where each point is counted according its multiplicity. We denote by $\overline{E}(a;f)$ the reduced form of $E(a;f)$. For $A \subset \mathbb{C}$ we denote by $n_A(r,a;f)$ the number of zeros of $f-a$, counted with multiplicities, which lie in $A \cap \{z : |z| < r\}$. We define $N_A(r,a;f)$ as follows

$$N_A(r,a;f) = \int_0^r \frac{n_A(t,a;f) - n_A(0,a;f)}{t} dt + n_A(0,a;f) \log r.$$

Let f and g be two nonconstant meromorphic functions. We say that f and g share the polynomial a CM(counting multiplicities) if $E(a;f) = E(a;g)$. Also we say that f and g share a IM(ignoring multiplicities) if $\overline{E}(a;f) = \overline{E}(a;g)$. For standard definitions and results we refer the reader to (Hayman, 1964).

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AN ENTIRE FUNCTION SHARING A LINEAR POLYNOMIAL WITH ITS LINEAR DIFFERENTIAL POLYNOMIALS.

IMRUL KAISH AND GOUTAM KUMAR GHOSH

ABSTRACT. In this paper we study the uniqueness of an entire function when it shares a linear polynomial with its linear differential polynomials.

1. INTRODUCTION, DEFINITIONS AND RESULTS

Let f be a noncostant meromorphic function defined in the open complex plane $\mathbb{C}$. The integrated counting function of poles of f is defined by

$$N(r,\infty;f)=\int_0^r \frac{n(t,\infty;f)-n(0,\infty;f)}{t}dt+n(0,\infty;f)\log r.$$

where $n(t,\infty;f)$ be the number of poles of f lying in $|z|\le r$, the poles are counted according to their multiplicities and $n(0,\infty;f)$ be the multiplicity of pole of f at origin.

For a polynomial $a=a(z)$, $N(r,a;f)$ $(\overline{N}(r,a;f))$ be the integrated counting function (reduced counting function) of zeros of $f-a$ in $|z|\le r$.

Let $A\subset\mathbb{C}$, we denote by $n_A(r,a;f)$ the number of zeros of $f-a$, counted with multiplicities, that lie in $\{z:|z|\le r\}\cap A$. The corresponding integrated counting function $N_A(r,a;f)$ is defined by

$$N_A(r,a;f)=\int_0^r \frac{n_A(t,a;f)-n_A(0,a;f)}{t}dt+n_A(0,a;f)\log r.$$

We also denote by $\overline{N}_A(r,a;f)$ the reduced counting functions of those zeros of $f-a$ that lie in $\{z:|z|\le r\}\cap A$.

Clearly if $A=\mathbb{C}$, then $N_A(r,a;f)=N(r,a;f)$ and $\overline{N}_A(r,a;f)=\overline{N}(r,a;f)$.

We denote by $E(a,f)$ the set of zeros of $f-a$ counted with multiplicities and by $\overline{E}(a,f)$ the set of distinct zeros of $f-a$.

For the standard definitions and notations of the value distribution theory authors suggest to see [1] and [8]

The investigation of uniqueness of an entire function sharing certain values with its derivatives was initiated by L. A. Rubel and C. C. Yang [7] in 1977. They proved the following result.

Theorem A. [7]. *Let f be a non-constant entire function. If $E(a;f)=E(a;f^{(1)})$ and $E(b;f)=E(b;f^{(1)})$, for distinct finite complex numbers a and b, then $f\equiv f^{(1)}$.*

In 1979 E.Mues and N.Steinmetz [6] took up the case of IM sharing in the place of CM sharing of values and proved the following theorem.

Theorem B. [6]. *Let f be a non-constant entire function and a, b be two distinct finite complex values and . If $\overline{E}(a;f)=\overline{E}(a;f^{(1)})$ and $\overline{E}(b;f)=\overline{E}(b;f^{(1)})$, then $f\equiv f^{(1)}$.*

The uniqueness of an entire function sharing a nonzero finite value with its first two derivatives was considered by G. Jank, E. Mues and L. Volkmann [2] in 1986. The following is their result.

Theorem C. [2]. *Let f be a nonconstant entire function and a be a nonzero finite value. If $\overline{E}(a;f)=\overline{E}(a;f^{(1)})\subset\overline{E}(a;f^{(2)})$, then $f\equiv f^{(1)}$.*

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Entire Functions Sharing a Second order Polynomial with its Derivatives

Goutam Kumar Ghosh
Dr. Bhupendranath Dutta Smriti Mahavidyalaya,
Hatgobindapur, Burdwan, India

Abstract: We prove a uniqueness theorem for an entire function, which share a function with their first and second order derivatives. We improve some existing results.
Keywords: Entire function, Polynomial, Uniqueness

1 Introduction, Definitions and Results

Let f be a non-constant meromorphic function in the open complex plane $\mathbb{C}$. We denote by $T(r,f)$ the Nevanlinna characteristic function of f and by $S(r,f)$ any quantity satisfying $S(r,f)=o\{T(r,f)\}$ as $r\to\infty$ except possibly a set of finite linear measure.

Let f and g be two non-constant meromorphic functions and let a be a complex number. We denote by $E(a;f)$ the set of a-points of f, where each point is counted according its multiplicity. We denote by $\overline{E}(a;f)$ the reduced form of $E(a;f)$. We say that f and g share a CM, provided that $E(a;f)=E(a;g)$, and we say that f and g share a IM, provided that $\overline{E}(a;f)=\overline{E}(a;g)$. In addition, we say that f and g share ∞ CM, if $\frac{1}{f}$ and $\frac{1}{g}$ share 0 CM, and we say that f and g share ∞ IM, if $\frac{1}{f}$ and $\frac{1}{g}$ share 0 IM.

For standard definitions and notations of the value distribution theory we refer the readers to [2]. However we require the following definitions.

Definition 1.1 A meromorphic function $a=a(z)$ is called a small function of f if $T(r,a)=S(r,f)$.

Definition 1.2 Let f and g be two non-constant meromorphic functions defined in $\mathbb{C}$. For $a,b\in\mathbb{C}\cup\{\infty\}$ we denote by $N(r,a;f\mid g\neq b)(\overline{N}(r,a;f\mid g\neq b)$ the counting function (reduced counting function) of those a-points of f which are not the b-points of g.

Definition 1.3 *Let f and g be two non-constant meromorphic functions defined in $\mathbb{C}$. For $a,b\in\mathbb{C}\cup\{\infty\}$ we denote by $N(r,a;f\mid g=b)(\overline{N}(r,a;f\mid g=b)$ the counting function (reduced counting function) of those a-points of f which are the b-points of g.*

In 1977 L.A.Rubel and C.C.Yang [7] first investigated the uniqueness of entire function sharing certain values with their derivatives. They proved the following result.

Theorem A *[7] Let f be a nonconstant entire function. If $E(a;f)=E(a;f^{(1)})$ and $E(b;f)=E(b;f^{(1)})$ for two distinct finite complex numbers a and b then $f\equiv f^{(1)}$.*

In 1979 E.Mues and N.Steinmetz [6] improved theorem A in the following manner.

Theorem B *[6] Let a and b be two distinct finite complex numbers and f be a nonconstant entire function. If $\overline{E}(a;f)=\overline{E}(a;f^{(1)})$ and $\overline{E}(b;f)=\overline{E}(b;f^{(1)})$, then $f\equiv f^{(1)}$.*